# THE GRAND CALCULUS

## Project History, Editorial Method, and Scope

A documentary essay on the complete modernized English edition of G. M. Fikhtengol'ts's *Course of Differential and Integral Calculus*

Biography • original publication history • scope of the 7+1-volume set
original-versus-modernized comparison • complete editorial prefaces

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Translator and editor
Toronto, Canada • 2026

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## The Grand Calculus: Project History, Editorial Method, and Scope

A documentary essay on the complete modernized English edition of G. M. Fikhtengol'ts's *Course of Differential and Integral Calculus*

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The complete Translator's Preface from the core volumes and the complete Introduction to Volume 8 are reproduced here by the translator and editor, who owns the copyright in their English text.

**REPOSITORY SCOPE**

This file is an original documentary article about the project. It does not contain the complete eight books and does not imply endorsement by the repository that hosts it.

**EDITION NOTE**

Archive article, August 2026. The bibliographic facts, comparative findings, and quantitative scope reported here were checked against the final LaTeX source of the eight-volume edition. Representative comparisons were also checked against digitized Russian editions. External historical claims are documented in the source notes.

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# Contents

**Project Overview** **1**
Abstract . . . 1
The project at a glance . . . 1
Fikhtengol'ts: a life in analysis . . . 2
From a prewar plan to the postwar Course . . . 2
Contemporary reception . . . 3
Scale, content, and international afterlife . . . 3
What the new edition contributes . . . 4
Basis of the original-to-modernized comparison . . . 4
From three Russian volumes to seven core volumes . . . 5
What modernization changes—and what it preserves . . . 6
Representative comparisons: modernization in practice . . . 6
Authorship and editorial responsibility . . . 8
Architecture of the 7+1-volume set . . . 9
The exercise companion in detail . . . 9
Recreated mathematical graphics . . . 9
How the collection may be used . . . 12
Rights, territories, and access . . . 12

**Part II: Complete Translator's Preface** **13**
**Translator's Preface** **14**
About "The Grand Calculus" . . . 14
The original Course and its legacy . . . 14
Recommended use . . . 15
Approach to translation . . . 16
Scope of the modernization . . . 16
Technology and editorial responsibility . . . 17
Territorial availability . . . 17
Acknowledgements . . . 17
Contents of the eight volumes . . . 18

**Part III: Complete Introduction to Volume 8** **20**
**Introduction to Volume 8** **21**
The companion problem volume . . . 21
How the volume is organized . . . 21
Territorial availability . . . 21
**About the Translator and Editor** **22**
**Selected References and Source Notes** **23**

# Project Overview


## Abstract

*The Grand Calculus* is a 2026 English-language publishing project built around Grigorii Mikhailovich Fikhtengol'ts's three-volume Russian classic, *A Course of Differential and Integral Calculus*. Its first seven volumes form a complete modernized English translation of the full *Course*, reorganized thematically into twenty-three chapters. An eighth, separately authored companion volume contains 2,798 exercises drawn from the classical problem collections of N. M. Günter and R. O. Kuzmin, coordinated with the notation, terminology, and sequence of the translated work and supplied with answer keys.

The project is intended to make a major tradition of rigorous mathematical analysis available to English-speaking readers as a working text rather than as an antiquarian facsimile. It preserves the original work's breadth, proofs, examples, geometric insight, and patient exposition while updating mathematical English, notation, organization, typography, graphics, formula numbering, and cross-references. The final sources contain 233 distinct figure assets across the core set; the Preface reports that the illustrations were recreated from their mathematical content rather than inserted as scans. Artificial intelligence assisted with technical and repetitive tasks, while the translator and editor retained responsibility for every substantive mathematical and editorial decision. The complete Preface below gives the project's full account of the human work involved.

This article places the project in Fikhtengol'ts's life, the history and reception of the 1947–1949 Russian *Course*, and its international afterlife. It maps the original three volumes to the seven-volume English core, compares a representative Russian passage with the modernization, and documents scope, editorial method, authorship, graphics, and intended use. It also reproduces in full the Translator's Preface used in the seven core volumes and the separate Introduction to Volume 8.


## The project at a glance

| ELEMENT | DESCRIPTION |
|---|---|
| Original work | G. M. Fikhtengol'ts, *A Course of Differential and Integral Calculus*, first published in three Russian volumes, 1947–1949. |
| Core edition | Seven volumes, twenty-three chapters, approximately 2,400 pages as described in the edition's Preface, and 233 distinct figure assets. |
| Companion | One exercise volume, nine chapters, 2,798 exercises, coordinated review notes, marked enrichment material, and answers to every exercise. |
| Editorial model | Complete modernized English translation: preservation of the original mathematical work with contemporary language, notation, navigation, graphics, and typography. |
| Translator and editor | Maksim Sokolov, Professor of Mathematics, Seneca Polytechnic, Toronto. |
| Publisher | Maxergon School, Toronto, Canada, 2026. |

### Fikhtengol'ts: a life in analysis

Grigorii Mikhailovich Fikhtengol'ts (1888–1959), whose surname also appears as Fikhtengolts and Fichtenholz, was born in Odessa. He finished gymnasium there in 1905 and graduated in 1911 from Imperial Novorossiisk University, where S. O. Shatunovsky was among his formative teachers. His early professional life already joined mathematics, exposition, and translation: at the Odessa publishing house Mathesis he translated Otto Dziobek's analytical geometry and assisted K. A. Posse with the Russian edition of a work by Ernesto Cesàro. In 1913 he moved to St. Petersburg and began teaching at the Imperial Electrotechnical Institute. He later worked with J. D. Tamarkin on the first Russian volume of Charles-Jean de la Vallée Poussin's *Cours d'analyse infinitésimale* (1922), and served as translator, annotator, and editor when the second volume appeared in 1933. [1]

His research of 1913–1917 on real-variable function theory, especially integrals depending on a parameter, led to the master's dissertation he defended in 1918. That same year he joined Petrograd University, later Leningrad University, where he taught for the rest of his life. His principal research areas were the theory of functions of a real variable and functional analysis, including absolute continuity, parameter-dependent integrals, Poisson–Stieltjes integrals, convergence in measure, linear operations, and function spaces. For many years he and L. V. Kantorovich led a functional-analysis seminar; the circle of students associated with his teaching included Kantorovich, I. P. Natanson, S. L. Sobolev, D. K. Faddeev, and S. A. Khristianovich. His legacy is therefore not confined to a famous textbook: it belongs to the formation of the Leningrad analytical school and to several generations of mathematical teaching. [2]

His educational work also extended beyond the university lecture room. Demidov and Petrova report that he founded Leningrad University's chair of mathematical analysis and directed it until 1953, when he was removed during the campaign against so-called cosmopolitanism; he nevertheless continued teaching at the university. He participated in state work on school and teacher-training mathematics programs, and in 1934 initiated the first Leningrad Mathematical Olympiad for schoolchildren. The *Course* belongs within this larger effort to build the institutions, curricula, and books through which analysis would be learned.

### From a prewar plan to the postwar *Course*

Fikhtengol'ts's mature *Course* emerged from decades of teaching and textbook work. Before it, he published *Mathematics for Technicians* (1926) and the three-part *Mathematics for Engineers* (1931–1933). The plan for a fundamental treatise on the calculus developed over many years. In 1939 Leningrad University Press issued the first part of his *Mathematical Analysis*, prepared from lectures he had repeatedly given to first-year mathematics students. Its Preface acknowledged lecture notes made by N. I. Feldman and V. P. Basov, and G. R. Lorentz served as editor. A planned second part did not appear after the Second World War interrupted the project. Only after the war could Fikhtengol'ts resume and reorganize it in definitive form. Gostekhizdat, the State Publishing House for Technical

[1] S. S. Demidov and S. S. Petrova, "G. M. Fikhtengol'ts and the Teaching of Mathematical Analysis in Russia during the First Half of the Twentieth Century," *Chebyshevskii Sbornik* 20:3 (2019), 437–452, bibliographic record and full text at Math-Net.Ru.

[2] L. V. Kantorovich and I. P. Natanson, "Grigorii Mikhailovich Fikhtengol'ts," *Uspekhi Matematicheskikh Nauk* 14:5(89) (1959), 123–128; bibliographic record and full text at Math-Net.Ru.

and Theoretical Literature, issued the three volumes of *A Course of Differential and Integral Calculus* in Moscow and Leningrad in 1947, 1948, and 1949. [3]

The timing matters. Russian university analysis had been absorbing the Weierstrassian reorganization of limits, continuity, and rigor while the Soviet higher-education system was rebuilding and expanding. The *Course* answered that institutional need without becoming a narrow syllabus. In genre it retained something of the large French *traité*: a connected exposition in which theory, calculations, historical remarks, geometry, and applications illuminate one another. Fikhtengol'ts aimed not only to establish results but to show how the major parts of classical analysis fit together.

## Contemporary reception

Volume I appeared as a 690-page book in a first printing of 25,000 copies. A 1948 review by V. V. Nemytskii is valuable precisely because it was neither a publisher's notice nor unqualified praise. Nemytskii questioned several choices—the older term *varianta* for an ordered variable, the limited emphasis on operators, and the treatment of some local-versus-global questions. Yet he judged the exposition highly accessible, praised the author's pedagogical and scientific command, singled out the physical and mechanical examples and several elegant treatments, and predicted wider recognition. The review shows a living textbook entering debate, not a classic canonized only in retrospect. [4]

Kantorovich and Natanson's 1959 obituary later assessed the complete *Course* as an exceptional exposition of mathematical analysis, emphasizing its completeness, rigor, clarity, literary quality, historical material, and its routes toward integral equations, real-function theory, and functional analysis. Those features explain why the Russian work is better described as a comprehensive course in classical analysis than as a conventional first calculus text.

## Scale, content, and international afterlife

| RUSSIAN VOL. | FIRST ISSUE | PRINCIPAL SCOPE |
|---|---|---|
| I | 1947 | Real numbers; limits; functions of one variable; derivatives and differentials; qualitative study of functions; multivariable differential calculus; Jacobians; geometry; extension of functions. |
| II | 1948 | Indefinite and definite integration; applications; numerical and functional series; improper integrals; parameter-dependent integrals; infinite products; asymptotic and divergent-series methods. |
| III | 1949 | Line, double, surface, triple, and multiple integrals; bounded variation and the Stieltjes integral; vector analysis; Fourier series, integrals, and transforms; a general theory of limits. |

This range explains the book's continued usefulness as a reference. Topics often separated into later courses—Euler–Maclaurin summation, infinite products, asymptotic expansions, the Stieltjes integral, Fourier transforms, and a generalized theory of limits—remain connected to the basic calculus that

[3] The publication story is documented by Demidov and Petrova, pp. 445–447. The original sequence is independently recorded in the bibliography accompanying Kantorovich and Natanson's 1959 obituary and in the Math-Net.Ru bibliographic database.

[4] V. V. Nemytskii, review of G. M. Fikhtengol'ts, *Kurs differentsial'nogo i integral'nogo ischisleniia*, vol. I, *Uspekhi Matematicheskikh Nauk* 3:4(26) (1948), 181–183; full text at Math-Net.Ru.

motivates them. Historical notes identify the development of ideas; detailed examples prepare readers for independent problem solving even though the *Course* itself is not organized as an exercise manual.

The work's afterlife is unusually visible. Complete or partial editions appeared in German, Polish, and Chinese. A Polish catalogue records a twelfth edition; Higher Education Press lists an eighth Chinese edition and describes the three-volume scope. Russian institutional catalogues list nineteenth stereotyped editions of Volumes I and II in 2025 and 2026. These records do not prove that every reader uses the book in the same way, but they do establish continuous publication across languages and generations. [5]

The *Course* is substantially larger than Fikhtengol'ts's later, separate two-volume *The Fundamentals of Mathematical Analysis*. Some Russian-speaking readers informally distinguish the complete work as the "Grand Fikhtengol'ts" (*Bol'shoi Fikhtengol'ts*). The English project title preserves that distinction; it does not claim that no shorter work by Fikhtengol'ts had previously appeared in English.

### What the new edition contributes

The project addresses two separate problems. First, the full three-volume *Course* had not previously been available in a complete English translation, although the shorter *Fundamentals* and selected adapted materials had appeared in English. Second, a literal reproduction of mid-twentieth-century terminology, notation, numbering, graphics, and organizational conventions would impose avoidable barriers on a contemporary reader.

The edition therefore follows a dual commitment: completeness in mathematical substance and modernization in presentation. The translator's structural comparison reports that approximately 99.9 percent of the original material is retained. The governing sequence of ideas, proofs, examples, and depth is preserved; only genuinely obsolete material is removed. Additions are used to clarify an argument, expose an implicit connection, repair a presentation difficulty, or align terminology and pedagogy with current practice.

Modernization extends to the architecture of the edition. The three original volumes are reorganized thematically into seven manageable core volumes. Section and formula numbering has been rebuilt, the cross-reference network reconstructed, and notation standardized. The Preface further reports that every included graph and diagram was recreated from its mathematical content rather than copied as a scan. The result is meant to function as a contemporary textbook and reference while remaining recognizably Fikhtengol'ts's work.

### Basis of the original-to-modernized comparison

The comparison in this article separates what can be checked directly from what is reported by the translator. The final 2026 LaTeX sources were checked against digitized Russian editions of Volume I (Nauka, 1969), Volume II (Fizmatgiz, 1959), and Volume III (Fizmatgiz, 1960). These scans are later Russian editions rather than the 1947–1949 first impressions, but they preserve the established Russian structure and provide a practical basis for checking the table of contents, section sequence, terminology, examples, and representative passages.[6]

The resulting comparison verifies the structural remapping and illustrates the editorial method; it is not a new line-by-line critical collation of roughly 2,400 English pages. The statement that approximately 99.9 percent of the original material is retained is the translator's project-wide structural

[5] See the Polish academic-library record, Higher Education Press catalogue, and Bauman Moscow State Technical University records for Volume I and Volume II.

[6] Digitized Russian comparison copies: Volume I, Volume II, and Volume III.

finding, reproduced in the Preface below. It should not be mistaken for an independent certification by Internet Archive or by the external historical sources cited here.

### From three Russian volumes to seven core volumes

| RUSSIAN DIVISION | GRAND CALCULUS DIVISION | CONTINUITY OF CONTENT |
|---|---|---|
| Volume I: real-number Introduction, Chapters I–VII, and the supplement "Extension of Functions" | Volumes 1–2; modern Chapters 1–9 | The unnumbered introduction becomes Chapter 1. The seven Russian chapters become Chapters 2–8, and the supplement (Sections 257–262) becomes Chapter 9. |
| Volume II: Chapters VIII–XIV | Volumes 3–5; modern Chapters 10–16 | Integral calculus, series, improper integrals, and parameter-dependent integrals retain their sequence but are separated into three usable thematic volumes. |
| Volume III: Chapters XV–XX and the supplement "General Point of View on Limit" | Volumes 6–7; modern Chapters 17–23 | Geometric and multiple integration form Volume 6. Fourier analysis forms Chapters 21–22, and the supplement (Sections 752–762) becomes Chapter 23, "General Theory of Limits." |
| Separate problem books | Volume 8, companion | The 2,798 exercises come from Günter and Kuzmin, not from Fikhtengol'ts's *Course*; they are coordinated with, but do not alter, the seven-volume theoretical text. |

### What modernization changes—and what it preserves

| FEATURE | RUSSIAN COURSE | 2026 EDITION |
|---|---|---|
| Mathematical sequence | A connected progression from the real numbers through limits, calculus, integration, series, multiple integration, and Fourier analysis. | The same progression, governing proofs, examples, and depth; physical volume boundaries change, but the conceptual order remains. |
| Language | Mid-twentieth-century Russian prose and terminology, including historically specific terms that do not map naturally into present mathematical English. | Idiomatic mathematical English; literal phrasing is not retained when it obscures meaning. Standard terms are used consistently across all seven volumes. |
| Definitions and notation | Rigorous prose definitions with the notation and expository habits of the Russian editions. | Modern symbols and set notation where useful, explicit quantifier forms in selected definitions, standardized $\mathbb{N}$ and $\mathbb{R}$, and added clarifications where older conventions could mislead. |
| Organization | Three large physical volumes: an unnumbered real-number introduction, twenty numbered chapters, and separate supplements on extension of functions and the general point of view on limit. | Seven thematic core volumes and twenty-three numbered chapters, followed by a separately authored exercise companion. |
| Navigation | Printed section and formula numbering tied to the original volumes; page-based and local references. | Rebuilt section and formula numbering, volume-aware references, hyperlinks, bookmarks, indexes, and a reconstructed cross-reference network. |
| Graphics | Printed line drawings integrated into successive Russian editions. | The final sources contain 233 distinct figure assets. The Preface reports that the illustrations were redrawn from their mathematical content with modern line work, resolution, and typography rather than inserted as scans; a direct sample comparison confirms this method for the Lagrange-remainder figure reproduced later in this article. |
| Editorial additions | Fikhtengol'ts's own exposition and historical notes. | Occasional explanations, terminology notes, and connective material support the source without being typographically separated into a second editorial layer. |
| Content retention | The governing Russian source. | The Preface reports approximately 99.9% structural retention; only material judged genuinely obsolete is omitted. This percentage is the translator's audit, not an external textual census. |
| Production method | Conventional twentieth-century print composition. | Born-digital LaTeX composition. AI assisted transcription, drafting, encoding, and consistency checks; Maksim Sokolov made and accepts responsibility for substantive mathematical and editorial decisions. |

### Representative comparisons: modernization in practice

The examples below are deliberately representative rather than exhaustive. They document visible editorial decisions at several scales; they do not imply that every sentence was altered, every theorem was re-proved, or the complete work was independently collated line by line for this article.

The opening of the theory of limits provides a concrete example. In Russian Volume I, Section 22 begins with the physical idea of a variable quantity, then introduces an ordered list and follows Charles Méray in calling a variable that assumes those values a *varianta*. Section 23 defines its limit by

the familiar epsilon condition. The 2026 edition preserves the examples—arithmetic and geometric progressions, perimeters of inscribed polygons, and decimal approximations to $\sqrt{2}$—and preserves the epsilon definition.

The presentation, however, is recast around the modern object: a numerical sequence. It distinguishes discrete from continuous variables, describes a sequence as an ordered list and, in a note, as a function $x : \mathbb{N} \to \mathbb{R}$. The limit definition is also displayed in quantifier notation, with the dependence $N = N_\varepsilon$ made explicit. The historical expression "infinitesimal" is retained, while "null sequence" and a warning against treating an individual nonzero term as an infinitesimal are supplied for present-day readers.

The opening foundations provide a larger example of added explanation. The contents of Russian Volume I proceed from Section 21, on measurement of segments, directly to Section 22 and the theory of limits. The 2026 edition retains the substance and order of those two original sections but expands the passage between them: after the material corresponding to Section 21 and before the material corresponding to Section 22, it inserts two new, separately titled bridge sections. The first defines countable and uncountable sets, enumerates $\mathbb{Z}$ and $\mathbb{Q}$, proves by Cantor's diagonal argument that $(0, 1)$ is uncountable, and records a bijection between $(0, 1)$ and $\mathbb{R}$. The second introduces partial and total orders, groups, fields, and the concise modern description of $\mathbb{R}$ as a complete Archimedean ordered field. These additions make abstract background that many present-day courses assume available at the point where it first becomes useful; they do not replace Fikhtengol'ts's construction of the real numbers.

The same comparison reveals additions at two different scales. In the original Volume II, Section 263 writes the indefinite integral as $\int f(x)\,dx = F(x) + C$. The modernization adds the explicit set interpretation $\{F + C : C \in \mathbb{R}\}$, so that later identities between indefinite integrals are not misread as equalities between single functions.

Chapter 23 provides the larger-scale example. The original Russian Volume III concluded with a nineteen-page general-limit supplement (pp. 631–649, Sections 752–762). The new edition retains that complete numbered sequence and its original line of argument, while inserting Section 752a between Sections 752 and 753. This preliminary section introduces metrics, balls, neighbourhoods, cluster points, topology, and equivalent metrics, together with the modern caveat that sequences do not detect every limiting process in arbitrary topological spaces. Across the retained sections, the historical theory of ordered variables is then connected explicitly with strict orders, preorders, directed sets, nets and Moore–Smith convergence, subnets, filters, Hausdorff spaces, and the net characterization of compactness. Selected results are stated with the hypotheses expected in modern analysis: for example, uniqueness of a net limit is tied to the Hausdorff condition, and the indiscrete topology supplies a counterexample when that condition is absent. Thus the chapter preserves Fikhtengol'ts's framework while developing it into a fuller contemporary introduction to generalized limits.

Modern notation is introduced where it gives a useful name to an idea already present in the classical discussion. In the treatment of global extrema, Section 139 adds set-valued definitions of arg max and arg min, distinguishing the points where extrema are attained from the extremal values themselves. Section 140 then retains the classical optimization applications and adds a statistical example: from $s$ successes in $n$ Bernoulli trials, differentiation of the binomial likelihood yields the maximum-likelihood estimate $s/n$. This illustrates the broader editorial policy reported in the Preface: material judged genuinely obsolete may be removed, while new examples are added when they connect the classical method to present-day language or applications.

Across the seven core volumes, the same method appears in smaller interventions. Historical or local names are replaced by—or paired with—the standard English terminology now used in mathematics; occasional notes add context, warnings, domain restrictions, or missing intermediate explanations; and selected theorem statements and proofs are tightened by making hypotheses and limiting operations explicit. All numbered formulas have been renumbered, section references have been rebuilt as a volume-aware system, and the entire cross-reference network has been reconstructed and checked. Together with redrawn figures, hyperlinks, bookmarks, and indexes, these changes modernize both the reading of an argument and the ability to verify where it is used elsewhere in the collection.

A source-level structural check found every inherited Section 1 through Section 762 exactly once across the modern core and no gap in that sequence; Section 752a is the single separately numbered insertion. This verifies the chapter-and-section remapping summarized above. It does not, by itself, establish the Preface's broader 99.9-percent retention estimate, which would require a line-by-line critical collation.

This sample shows the governing editorial pattern. The mathematics, examples, and argumentative route remain recognizable; the conceptual language, notation, and local explanation are rebuilt so that an English-speaking reader meets the material in the idiom of current analysis. The edition is therefore neither a facsimile nor a word-for-word documentary translation. It is a working modernization whose source relationship is stated openly.

### Authorship and editorial responsibility

| PERSON OR IMPRINT | ROLE | SCOPE |
|---|---|---|
| G. M. Fikhtengol'ts | Original author | Governing author of Volumes 1–7, which translate and modernize the full *Course*. |
| Maksim Sokolov | Translator and editor | English translation, editorial work, mathematical review, typesetting, design, and final responsibility for Volumes 1–7. |
| N. M. Günter and R. O. Kuzmin | Source authors for Volume 8 | Authors of the classical problem collections from which the companion exercises were selected and adapted. Volume 8 is not part of Fikhtengol'ts's original *Course*. |
| Maksim Sokolov | Translator and editor of Volume 8 | Selection, translation, reorganization, adaptation, coordination with the core volumes, and typesetting. |
| Maxergon School | Publisher | Publisher and imprint of the 2026 English edition. |

Artificial intelligence was used as a technical instrument for preliminary drafts, transcription, LaTeX encoding, cross-reference checking, consistency checking, and other repetitive tasks. It did not determine the meaning of the translation, the validity of a proof, the structure of the edition, or the substance of additions and omissions. As the complete Preface below explains, substantive judgment and responsibility remain with the translator and editor.

### Architecture of the 7+1-volume set

| VOL. | TITLE | CHAPTERS | ISBN |
|---|---|---|---|
| 1 | *Real Numbers, Limits, and Differential Calculus* | 1–5 | 978-1-7380649-3-9 |
| 2 | *Multivariable Differential Calculus and Geometry* | 6–9 | 978-1-7380649-4-6 |
| 3 | *Methods and Applications of Integral Calculus* | 10–12 | 978-1-7380649-5-3 |
| 4 | *Numerical and Functional Series* | 13–14 | 978-1-7380649-6-0 |
| 5 | *Improper and Parameter-Dependent Integrals* | 15–16 | 978-1-7380649-7-7 |
| 6 | *Line, Surface, and Multiple Integrals* | 17–20 | 978-1-0679668-1-2 |
| 7 | *Fourier Series, Integrals, and Transforms* | 21–23 | 978-1-7380649-9-1 |
| 8 | *Exercises* (companion) | 1–9 local | 978-1-0679668-0-5 |

The chapter descriptions in the complete Preface below show how the sequence moves from real numbers and limits through one- and several-variable differential calculus; indefinite, definite, improper, and parameter-dependent integration; series; line, surface, and multiple integration; Fourier analysis; and a general theory of limits using directed orders, nets, filters, and generalized limit superior and inferior.

### The exercise companion in detail

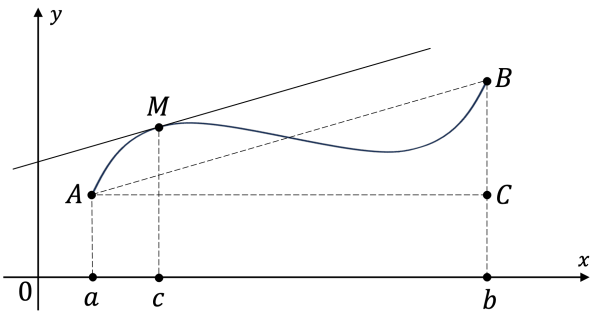
Volume 8 is a coordinated practice book rather than a continuation of Fikhtengol'ts's theoretical exposition. Each section points back to the relevant core chapters. Problems that extend beyond the core presentation are marked as enrichment and supply the local definitions or formulas needed to attempt them. Answers are provided for all 2,798 exercises; they are answer keys, not a claim of complete worked solutions in every case.

| CHAPTER | TOPIC | EXERCISES |
|---|---|---|
| 1 | Limits | 168 |
| 2 | Differentiation | 417 |
| 3 | Applications of Differential Calculus to Analysis | 429 |
| 4 | Indefinite Integrals | 460 |
| 5 | Definite Integrals and Their Applications | 206 |
| 6 | Multiple, Line, and Surface Integrals | 365 |
| 7 | Advanced Definite Integrals | 414 |
| 8 | Infinite Series | 234 |
| 9 | Numerical Integration and Series Methods | 105 |
| | **Total** | **2,798** |

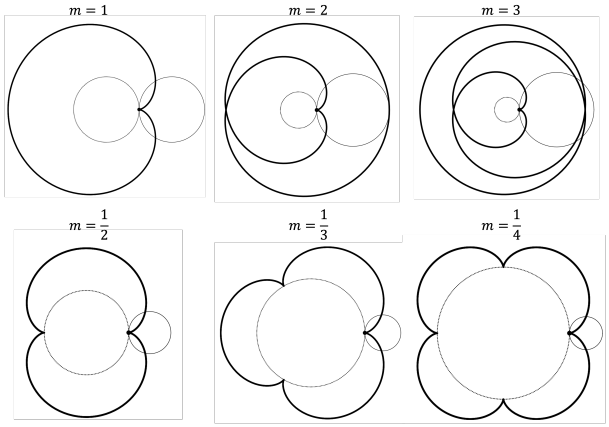
### Recreated mathematical graphics

The original illustrations were not inserted as scans. They were redrawn from their mathematical content for consistent line weight, labeling, resolution, and typographic integration. The seven examples on the following two pages range from foundational differential calculus to space geometry,

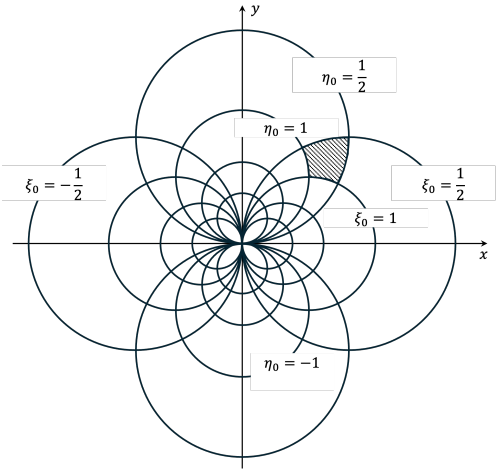
surface integration, heat flow, Fourier analysis, and engineering applications. The second plate deliberately favors denser figures drawn from several parts of the collection.



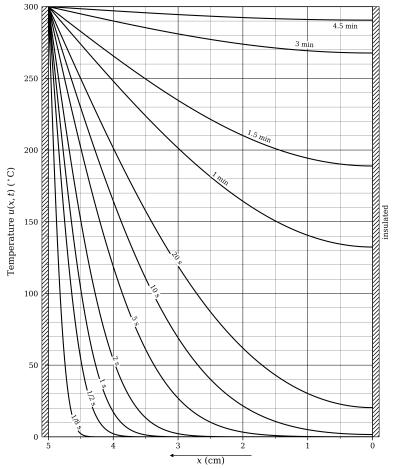

**A.** Geometric interpretation of Lagrange's mean-value theorem (Volume 1).




**B.** A family of epicycloids (Volume 2).

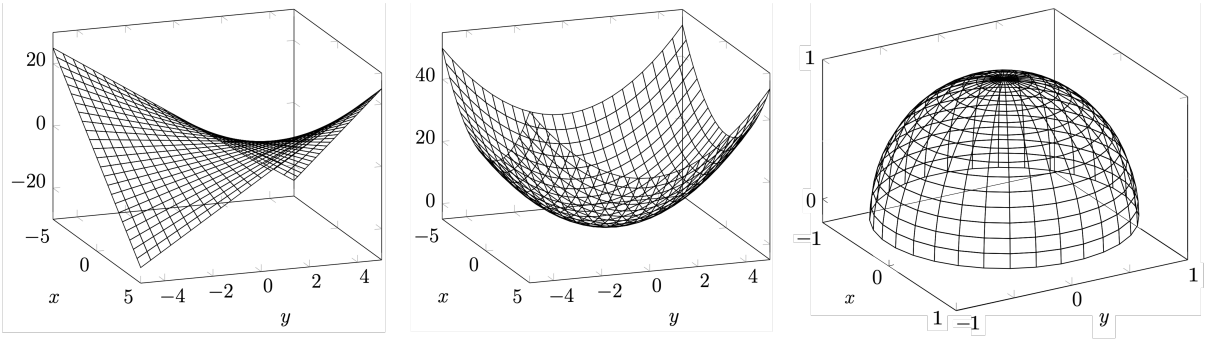



**C.** Coordinate curves in a change-of-variables example (Volume 6).

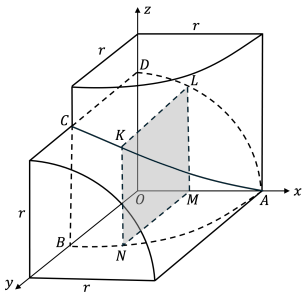



**D.** Temperature profiles from a Fourier-series heat solution (Volume 7).

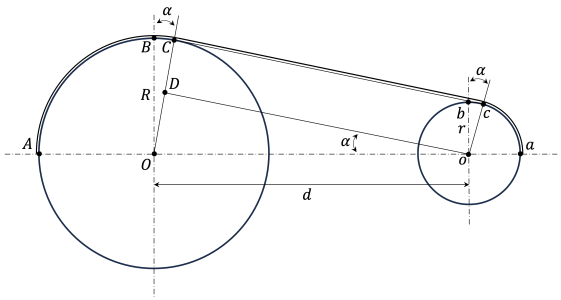
**Figure 1:** *Representative figures recreated for the modernized edition.*




**F.** Three spatial graphs of two-variable functions: a hyperbolic paraboloid, a paraboloid of revolution, and a hemisphere (Volume 2).




**E.** One octant of a Steinmetz solid formed by perpendicular cylinders (Volume 3).




**H.** Belt geometry around unequal pulleys in an engineering approximation (Volume 1).

**Figure 2:** *More intricate examples from space geometry, multiple integration, and engineering mechanics.*

### How the collection may be used

- **As a principal course or self-study text.** The exposition is rigorous and comprehensive, with motivations, intermediate arguments, examples, and geometric interpretation supplied at a scale unusual in more compressed treatments.
- **As a companion to a concise modern analysis text.** A reader using Rudin, Zorich, or another economical text may turn to Fikhtengol'ts for detailed calculations, figures, examples, and alternate routes into the same ideas.
- **As a reference.** The thematic seven-volume division and rebuilt cross-reference system allow a reader to consult a topic without carrying the entire original three-volume physical set.
- **For practice.** Volume 8 follows the core notation and sequence, making it possible to move directly from a theoretical chapter to a large, graded problem set and then to its answer key.

Accessibility should not be confused with elementary level. The collection is written to explain difficult mathematics clearly, not to reduce its demands. It is best suited to serious undergraduate and graduate students, instructors, mathematicians, and independent readers seeking a broad, classical treatment of analysis.

### Rights, territories, and access

The edition's July 2026 territorial statement identifies the underlying Russian works as public domain in Canada, Japan, New Zealand, India, South Africa, the Hong Kong Special Administrative Region, and the Republic of Korea, and states that the complete 7+1-volume edition is intended for distribution in those jurisdictions. That statement is reproduced verbatim in both primary-text sections below.

The 2026 English translation, adaptation, modernization, editorial material, typesetting, figures, and design are separately copyrighted by Maksim Sokolov and Maxergon School. The public-domain status of an underlying work in a particular jurisdiction does not place this new English edition in the public domain. This article describes the project for an international audience; availability of the complete books remains subject to the territorial notice and applicable law. Readers may use the ISBN table above to identify authorized editions where distribution is available.

As the necessary rights are lawfully obtained for additional territories, the volumes will be made available in those territories through authorized channels. No availability in a territory is implied before that legal clearance is complete.

PART II

# Complete Translator's Preface

Reproduced in full from Volume 1

The Preface's prose and footnotes are reproduced unabridged. Its eight-volume contents overview is presented as a neutral project-wide list, without a volume-specific label or frame.

# Translator's Preface

### About "The Grand Calculus"

The seven-volume set now before the reader, *The Grand Calculus*, is a complete, modernized English translation of G. M. Fikhtengol'ts's three-volume masterpiece, *A Course of Differential and Integral Calculus* (hereafter, the *Course*)[7]. The edition makes two contributions at once. It brings the full *Course* to English-speaking readers and presents the work in a form fully compatible with contemporary mathematical study. Translation accomplishes the first; modernization, the second. Neither has been achieved by reducing or simplifying the original. In its present form, the core seven-volume set spans around 2,400 pages (plus the eighth companion volume of approximately 500 pages). *The Grand Calculus* preserves the scale, substance, depth, progression of ideas, proofs, examples, and distinctive expository character of the original. Its presentation, notation, organization, illustrations, and system of cross-references have been modernized to bring the work into line with contemporary standards of calculus exposition.

This modernization builds on a work that has never ceased to be useful. More than seventy-five years after its appearance, the Russian-language *Course* remains in active use as both a textbook and a reference work—perhaps the strongest testimony to its enduring mathematical and pedagogical power. Yet, inexplicably, the *Course* has never before been available in a complete English translation. A translation was therefore necessary to make it accessible to English-speaking students and instructors. Modernization was equally necessary: certain aspects inevitably required revision.

The original three Russian volumes of the *Course* have been reorganized as seven core volumes of *The Grand Calculus*. The boundaries of these seven volumes were dictated by the nature of the subject matter, making each volume convenient for reading, teaching, and reference. The format has changed; the mathematical identity of the work has not.

The core seven volumes of *The Grand Calculus* are accompanied by the eighth companion volume devoted entirely to problems and exercises (it contains 2,798 exercises). Unlike the first seven volumes, which present Fikhtengol'ts's exposition of mathematical analysis, this companion volume draws upon the celebrated problem collections of Günther and Kuzmin, whose carefully graded and mathematically rich exercises have become a classic resource for generations of students of analysis. The problems have been selected, translated, reorganized, and adapted to follow the notation, terminology, and sequence of topics adopted throughout *The Grand Calculus*.

### The original *Course* and its legacy

Fikhtengol'ts wrote two major textbooks of mathematical analysis: the full three-volume *Course* and the shorter two-volume *The Fundamentals of Mathematical Analysis*. Generations of Russian-speaking students referred to the larger work informally as the "Grand Fikhtengol'ts" (*Bol'shoi Fikhtengol'ts*), in contrast with the shorter text. The title *The Grand Calculus* carries that traditional name into English and identifies this edition with the full three-volume work.

[7] More precisely, this is a "$7 + 1$"-volume set: the core seven volumes comprising the complete translation and modernization of Fikhtengol'ts's *Course*, together with the eighth companion volume containing a carefully selected and adapted collection of exercises drawn from the classic problem books of Günther and Kuzmin. See below in the Preface for a fuller explanation.

The stature of the *Course* has long been recognized. L. Kantorovich and I. Natanson described[8] it as "... one of the best expositions of mathematical analysis in the world literature, in terms of completeness, rigor, clarity, and literary skill."

S. Demidov and S. Petrova wrote[9]: "... this three-volume work, which has undergone numerous editions, holds an honorable place on the bookshelves of mathematicians—they turn to it to grasp the nuances of various topics in analysis or simply to refresh them in memory, and often to delve into questions that have hitherto remained on the periphery of their interests."

The *Course* was translated into many languages and used by generations of students in many countries. It is still used today, not merely as a historical classic but as a working textbook and reference, often alongside other books. Its durability rests on a rare combination of rigor, completeness, geometric insight, abundant examples and illustrations, and a patient, friendly manner of exposition. Fikhtengol'ts supplies motivations and intermediate arguments that more compressed texts often leave to the reader. The result is mathematically substantial yet unusually readable; at its best, the sustained unfolding of the subject gives the book the continuity of a mathematical novel.

As has already been mentioned, until this edition, the full *Course* had never been available in English. Fikhtengol'ts's shorter *The Fundamentals of Mathematical Analysis* did appear in English, but it is a substantially abridged and essentially different work. *The Grand Calculus* fills this longstanding gap.

### Recommended use

The accessibility of Fikhtengol'ts should not be mistaken for an elementary or entry-level treatment. The work is rigorous, comprehensive, and demanding. It is easier to follow because it explains more, not because it asks less. *The Grand Calculus* is substantial enough to serve as a principal text, but one of its greatest strengths is as a companion to a modern primary textbook, especially one whose treatment is concise or abstract.

A particularly natural pairing is with Walter Rudin's *Principles of Mathematical Analysis*[10] or V. A. Zorich's two-volume *Mathematical Analysis*.[11] Both present analysis from a modern, rigorous, and conceptually unified perspective, but their abstract and economical style can make them demanding on a first reading. Fikhtengol'ts supplies the detailed calculations, figures, motivations, numerous examples, and intermediate steps that allow the reader to see the theory in action.

For students wishing to strengthen their mastery through additional practice, the most natural companion is the eighth volume of *The Grand Calculus*, a dedicated collection of problems designed to follow the notation, terminology, and system of cross-references of the seven-volume main work. Readers seeking even more exercises will also find two classic resources particularly valuable: B. P. Demidovich's *Problems in Mathematical Analysis*[12] and W. J. Kaczor and M. T. Nowak's three-volume

[8] L. V. Kantorovich and I. P. Natanson, "Grigorii Mikhailovich Fikhtengol'ts," *Russian Mathematical Surveys*, 14:5(89) (1959), 123–128 (in Russian).

[9] S. S. Demidov and S. S. Petrova, "G. M. Fikhtengol'ts and the Teaching of Mathematical Analysis in Russia during the First Half of the Twentieth Century," *Chebyshevskii Sbornik*, 20:3 (2019), 437–452 (in Russian).

[10] W. Rudin, *Principles of Mathematical Analysis*, 3rd ed. (McGraw–Hill, 1976).

[11] V. A. Zorich, *Mathematical Analysis I*, 2nd English ed., translated by Roger Cooke and Octavio Paniagua Taboada, Universitext (Springer, 2015); and *Mathematical Analysis II*, 2nd English ed., translated by Roger Cooke and Octavio Paniagua Taboada, Universitext (Springer, 2016).

[12] B. P. Demidovich, *Problems in Mathematical Analysis*, translated from the Russian by G. Yankovsky (Mir Publishers, Moscow, 1970).

*Problems in Mathematical Analysis*.[13] Together, these works provide extensive collections of exercises that complement the present edition exceptionally well.

### Approach to translation

The translation of a classical mathematical work involves a choice of purpose.

One may produce a documentary translation, preserving the wording and conventions of the original as closely as possible. Such a translation has historical and documentary value, but it must also preserve terminology, notation, organization, and expository habits that may have become obstacles to present-day readers.

Alternatively, one may preserve the mathematics, structure, and authorial character of the work while allowing it to speak in contemporary mathematical English.

*The Grand Calculus* follows the second approach. It is not a word-for-word or line-by-line rendering. Literal phrasing was not retained when it would sound foreign, obscure the mathematics, or reproduce conventions that no longer serve a useful purpose. At the same time, modernization was governed by a strict principle: it must improve the work without simplification or abridgment. The original *Course* remains the governing source.

A legitimate alternative would be to mark every altered or added passage, allowing readers to distinguish the original text from each editorial intervention. I considered that method but did not adopt it. Modernization occurs throughout the work—in wording, terminology, notation, numbering, figures, cross-references, explanations, and occasional additions. Marking every instance would interrupt the mathematical flow on page after page and turn a working textbook into a layered critical edition. The English text is therefore presented as a single coherent whole. Fikhtengol'ts's original governs the core seven volumes throughout, but the translation and modernization are not typographically separated.

### Scope of the modernization

The following discussion concerns the modernization of Fikhtengol'ts's *Course*, which forms the core seven-volume work of *The Grand Calculus*.

The governing rule was preservation, not compression. This is not an abridged edition. A structural comparison with the Russian original shows that approximately 99.9% of the original material is present. The sequence of ideas, principal proofs, examples, and characteristic depth have been retained. Only material that had become genuinely obsolete was removed. Additions were made where they clarify an argument, make an implicit connection explicit, resolve a difficulty in presentation, or bring the treatment into line with contemporary terminology, notation, and pedagogy. Longstanding comments voiced by readers and mathematicians in online forums were also considered. The result remains overwhelmingly Fikhtengol'ts; the added material supports the original rather than displacing it.

The seven-volume organization of the *Course* is thematic rather than mechanical. The section-numbering system was rebuilt for easier navigation. All numbered formulas were renumbered according to the modern standard, and the entire network of cross-references was reconstructed and checked. I redrew graphs, diagrams, and illustrations from the mathematical content of the original using modern graphics technology. The old images were not simply scanned: they were recreated for

[13] W. J. Kaczor and M. T. Nowak, *Problems in Mathematical Analysis I: Real Numbers, Sequences and Series*, Student Mathematical Library, Vol. 4 (American Mathematical Society, 2000); *Problems in Mathematical Analysis II: Continuity and Differentiation*, Student Mathematical Library, Vol. 12 (American Mathematical Society, 2001); *Problems in Mathematical Analysis III: Integration*, Student Mathematical Library, Vol. 35 (American Mathematical Society, 2003).

a contemporary digital edition. Terminology and notation were standardized, and the typography was designed for comfortable sustained reading.

This modernization is extensive wherever the original had become dated and deliberately restrained elsewhere.

### Technology and editorial responsibility

The revolution in artificial intelligence (AI) made a project of this scale feasible for a single translator and convinced me that the time had come to undertake it. AI was indispensable as a technical instrument: it assisted with preliminary drafts, transcription and LaTeX encoding, cross-reference and consistency checks, and many other repetitive tasks. Without these tools, the complete edition could not realistically have been produced in this form by one person.

AI, however, did not replace mathematical or editorial judgment. Even with its assistance, the edition required three years of sustained, almost daily human work to review, correct, consolidate, rewrite, verify, and harmonize the whole. I made every substantive decision concerning meaning, terminology, proof, organization, addition, omission, and presentation. Human mathematical attention governed every stage, and responsibility for the final text is mine.

### Territorial availability

As of July 2026, the underlying Russian-language works on which the present "$7+1$"-volume edition is based—Fikhtengol'ts's *Course*, forming the core seven-volume work, and the Günther–Kuzmin problem collections, forming the eighth companion volume—are in the public domain in Canada, Japan, New Zealand, India, South Africa, the Hong Kong Special Administrative Region, and the Republic of Korea. Accordingly, the complete "$7+1$"-volume edition of *The Grand Calculus* is presently intended for distribution in the jurisdictions listed above.

### Acknowledgements

I am deeply grateful to Professors Svetlana Nekhai (Seneca Polytechnic) and Almaz Butaev (University of the Fraser Valley) for their thoughtful feedback, valuable suggestions, and professional insight.

Maksim Sokolov
Toronto, Canada
July 2026.

### Contents of the eight volumes

The "7+1"-volume structure follows the thematic development of the *Course*:

**Volume 1** [Chapters 1–5]

**Chapter 1. Real Numbers.** Develops the real-number system from rational and irrational numbers, including order, arithmetic operations, completeness properties, and decimal representations.

**Chapter 2. Theory of Limits.** Introduces sequences and limits, limit laws, monotone convergence, subsequences, and the Bolzano–Cauchy theory.

**Chapter 3. Functions of One Variable.** Develops functions, functional limits, infinitesimal and infinite quantities, continuity, discontinuities, and the principal properties of continuous functions.

**Chapter 4. Derivatives and Differentials.** Introduces derivatives and differentials, the fundamental mean-value theorems, higher derivatives, Taylor's formula, and interpolation.

**Chapter 5. Studying Functions Using Derivatives.** Uses derivatives to study monotonicity, extrema, convexity, graph shape, indeterminate forms, and approximate solutions of equations.

**Volume 2** [Chapters 6–9]

**Chapter 6. Functions of Several Variables.** Develops limits, continuity, partial derivatives, differentials, higher-order derivatives, and extrema for functions of several variables.

**Chapter 7. Functional Determinants and Their Applications.** Studies Jacobians and functional determinants, implicit and inverse functions, and changes of variables.

**Chapter 8. Applications of Differential Calculus to Geometry.** Applies differential calculus to curves and surfaces, including tangents, tangent planes, contact, arc length, and curvature.

**Chapter 9. Extension of Functions.** Develops smooth extensions of functions from intervals and planar regions, including boundary regularity and applications of extension theorems.

**Volume 3** [Chapters 10–12]

**Chapter 10. Indefinite Integral.** Presents methods of indefinite integration for rational, radical, trigonometric, exponential, and elliptic expressions.

**Chapter 11. Definite Integral.** Constructs the definite integral, proves its principal properties, develops evaluation and transformation methods, and introduces numerical quadrature.

**Chapter 12. Applications of Integral Calculus.** Applies integration to arc length, area, volume, mechanics, physics, engineering, and introductory differential equations.

**Volume 4** [Chapters 13–14]

**Chapter 13. Infinite Series with Constant Terms.** Develops convergence theory for numerical series, double series, infinite products, function expansions, approximations, and summation methods.

**Chapter 14. Functional Sequences and Series.** Studies sequences and series of functions, uniform convergence, power series, elementary complex functions, and asymptotic series.

**Volume 5** [Chapters 15–16]

**Chapter 15. Improper Integrals.** Develops improper integrals over unbounded intervals and for unbounded integrands, together with convergence tests, transformations, and approximations.

**Chapter 16. Integrals Depending on a Parameter.** Studies parameter-dependent integrals, uniform convergence, differentiation and integration under the integral sign, and the Beta and Gamma integrals.

**Volume 6** [Chapters 17–20]

**Chapter 17. Line Integrals and the Stieltjes Integral.** Develops line integrals of both kinds, path independence and potentials, functions of bounded variation, and the Stieltjes integral.

**Chapter 18. Double Integrals.** Develops double integrals, repeated integration, Green's formula, changes of variables, and improper double integrals.

**Chapter 19. Surface Area and Surface Integrals.** Treats two-sided surfaces, surface area, and surface integrals of the first and second kinds.

**Chapter 20. Triple and Multiple Integrals.** Develops triple and multiple integrals, the Gauss–Ostrogradsky theorem, changes of variables, and elements of vector analysis.

**Volume 7** [Chapters 21–23]

**Chapter 21. Fourier Series.** Introduces Fourier coefficients and series, convergence and approximation, Fourier integrals and transforms, and applications to differential equations and harmonic analysis.

**Chapter 22. Fourier Series (Continued).** Continues Fourier theory with completeness, mean-square approximation, summability methods, termwise operations, uniqueness, and the Dirichlet problem.

**Chapter 23. General theory of limits.** Recasts limits using directed orders and nets, including subnets, cluster values, filters, and generalized limit superior and limit inferior.

**Volume 8** [Chapters 1-9 of the companion volume]

**Exercises in Calculus with Answer Keys.**

PART III

# Complete Introduction to Volume 8

Reproduced in full from the exercise companion

# Introduction to Volume 8

### The companion problem volume

The first seven volumes of *The Grand Calculus* present the complete modernized English translation of G. M. Fikhtengol'ts's *A Course of Differential and Integral Calculus*. This eighth volume is their practical companion. Its 2,798 exercises have been selected and adapted from the classical problem collections of N. M. Günter and R. O. Kuzmin so that readers may consolidate the theory, acquire technical fluency, and test their command of the principal methods of mathematical analysis.

The present volume does not continue the systematic theoretical exposition of Fikhtengol'ts and is not part of his original *Course*. It is designed instead as an organic extension of the seven-volume work. The selection, wording, notation, terminology, order of topics, and review references have therefore been coordinated with that work throughout.

### How the volume is organized

The problems are arranged in nine chapters, beginning with limits and differentiation and proceeding through integration, multiple and vector integration, advanced integral and Fourier methods, infinite series, and numerical and asymptotic methods. Every section begins with a short review note identifying the volumes and chapters in the main work that contain the theory used by its problems. A reader should consult those chapters before attempting the section, or return to them when a method or hypothesis is no longer familiar.

Some exercises deliberately extend the theory developed in Volumes 1–7. They are marked **Enrichment** at the section or exercise level. Such a mark means that the topic is not assumed to have been studied in the main volumes; the definitions, formulas, and brief preliminary facts needed to attempt it are supplied locally in this volume. The review note continues to identify only the prerequisite chapters in Volumes 1–7.

**Chapter-numbering convention.** The chapters of Volume 8 are numbered as Chapters 1, 2, 3, and so on. Consequently, a chapter number that appears without a volume number is local to the volume in which it occurs. Every review note in this companion volume gives both the volume and chapter number when referring to the main work.

### Territorial availability

As of July 2026, the underlying Russian-language works on which the present "$7+1$"-volume edition is based—Fikhtengol'ts's *Course*, forming the core seven-volume work, and the Günther–Kuzmin problem collections, forming the eighth companion volume—are in the public domain in Canada, Japan, New Zealand, India, South Africa, the Hong Kong Special Administrative Region, and the Republic of Korea. Accordingly, the complete "$7+1$"-volume edition of *The Grand Calculus* is presently intended for distribution in the jurisdictions listed above.

# About the Translator and Editor

Maksim Sokolov is a Professor of Mathematics at Seneca Polytechnic in Toronto, Canada. He holds a PhD and a master's degree in mathematics, as well as a Master of Education in postsecondary mathematics education, and has taught mathematics in higher education since 2005.

In *The Grand Calculus*, Sokolov is the translator and editor of the seven-volume English edition. He also selected, translated, reorganized, adapted, and typeset the eighth exercise companion.

# Selected References and Source Notes


1. G. M. Fikhtengol'ts, *Kurs differentsial'nogo i integral'nogo ischisleniia* [*A Course of Differential and Integral Calculus*], 3 vols. (Moscow–Leningrad: Gostekhizdat, 1947–1949).

2. Digitized Russian comparison editions: Volume I, 7th ed. (Moscow: Nauka, 1969), Internet Archive item i-1969; Volume II (Moscow: Fizmatgiz, 1959), item ii-1959; and Volume III (Moscow: Fizmatgiz, 1960), item B-001-014-346.

3. L. V. Kantorovich and I. P. Natanson, "Grigorii Mikhailovich Fikhtengol'ts," *Uspekhi Matematicheskikh Nauk* 14:5(89) (1959), 123–128. Bibliographic record and full text: Math-Net.Ru.

4. S. S. Demidov and S. S. Petrova, "G. M. Fikhtengol'ts and the Teaching of Mathematical Analysis in Russia during the First Half of the Twentieth Century," *Chebyshevskii Sbornik* 20:3 (2019), 437–452. Math-Net.Ru.

5. V. V. Nemytskii, review of G. M. Fikhtengol'ts, *Kurs differentsial'nogo i integral'nogo ischisleniia*, vol. I, *Uspekhi Matematicheskikh Nauk* 3:4(26) (1948), 181–183. Math-Net.Ru.

6. Institutional records documenting later editions and translations: Bauman Moscow State Technical University, Volume I; Volume II; Polish academic-library record; and Higher Education Press, Chinese Volume I.

7. G. M. Fikhtengol'ts, *The Fundamentals of Mathematical Analysis*, 2 vols. (Oxford: Pergamon Press, 1965). This earlier English work is the shorter *Fundamentals*, not the complete three-volume *Course*.

8. G. M. Fikhtengol'ts, *The Grand Calculus*, Vols. 1–7, English translation and editorial work by Maksim Sokolov (Toronto: Maxergon School, 2026), with N. M. Günter and R. O. Kuzmin, *The Grand Calculus*, Vol. 8: *Exercises*, selected, translated, reorganized, and adapted by Maksim Sokolov (Toronto: Maxergon School, 2026).


#### NAMES AND TRANSLITERATION

The form *Fikhtengol'ts* is used throughout this edition. Library catalogues and earlier translations may use *Fichtenholz* or *Fikhtengolts*; all three forms identify the same author. The companion title page uses the spelling *Günter*; the core-volume Preface reproduces the variant *Günther*.

#### SUGGESTED CITATION

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